The Evolutionary Dynamics of Incubation Periods

Bertrand Ottino-Löffler
Steve Strogatz, Jacob Scott

10/06/19

The Incubation Period

Definition

The **Incubation Period** of a disease is defined to be the time between first exposure to a contagion and observation of first symptoms.

The Incubation Period


RICHMOND TIMES-DISPATCH: SUNDAY, JULY 11, 1915.

ed Danger to Everybody's Health – and No Remedy for It.

A 181.4

B 82.4

C 118.4

y Proof That a Dish of Baked Spaghetti "Culture" for Typhoid Germs. The Three and C, Indicate the Temperatures of the n Removed from the Oven—One-Half Inch Middle and Near the Bottom. The Typhoid Been Placed in the Mass Before Cooking

Survived in a Few Colonies Less Than an Inch Below the Baked Surface, While in the Centre of the Dish, with Its Mild Temperature of 82.4 Degrees, the Colonies Were Abundant and Active.

How a Dish of Baked Spaghetti Gave 93 Eaters Typhoid Fever

By Wilbur A. Sawyer, M. D.

to wash their hands disinfectant. The family ical directions how to will not convey germs

authorities is to detect h "sanitary isolation." quarantine, as for

The Incubation Period

NINETY-THREE PERSONS INFECTED BY A TYPHOID CARRIER AT A PUBLIC DINNER

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BERKELEY, CAL.

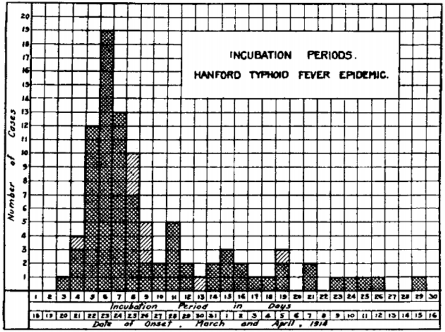


Chart of the cases in the Hanford typhoid fever epidemic, showing incubation periods and dates of onset. The heavily shaded areas represent definite cases of typhoid fever. The lightly shaded areas represent the doubtful cases.

The Incubation Period

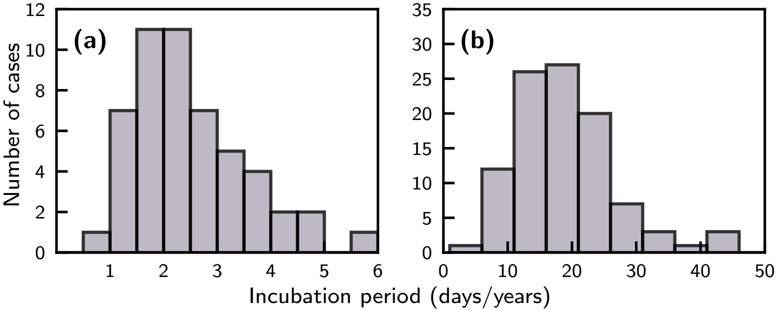
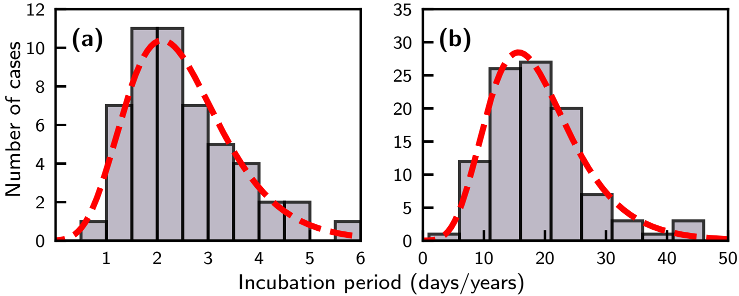


Figure: (a) Food-borne streptococcal sore throat (Sartwell 1950).
(b) Bladder tumors in a dye plant (Goldblatt 1949).

Sartwell's Law (1966)

Sartwell's Law

Incubation periods for diseases tend to be distributed as lognormals; more generally, they will be right-skewed.



An Evolutionary Graph Theory Is The Common Factor

The Illness …	… **Takes over the** …	… **Which is a** …
Typhoid	well-mixed gut microbiome	Complete graph
Leukemia	healthy bone marrow cells	3D lattice
Influenza	uncompromised tracheal cells	2D lattice

Yes, viruses.

Viruses **2018**, *10*(11), 627; https://doi.org/10.3390/v10110627 Open Access

Review

Causes and Consequences of Spatial Within-Host Viral Spread

by Molly E. Gallagher [1], Christopher B. Brooke [2,3], Ruian Ke [4] and Katia Koelle [1,*]

1 Department of Biology, Emory University, Atlanta, GA 30322, USA

Virus-Cell Interactions

Influenza A Virus Uses Intercellular Connections To Spread to Neighboring Cells

Kari L. Roberts, Balaji Manicassamy, Robert A. Lamb

D. S. Lyles, Editor

DOI

Influenza virus exploits tunneling nanotubes for cell-to-cell spread

Amrita Kumar, Jin Hyang Kim, Priya Ranjan, Maureen G. Metcalfe, Weiping Cao, Margarita Mishina, Shivaprakash Gangappa, Zhu Guo, Edward S. Boyden, Sherif Zaki, Ian York, Adolfo García-Sastre, Michael Shaw & Suryaprakash Sambhara

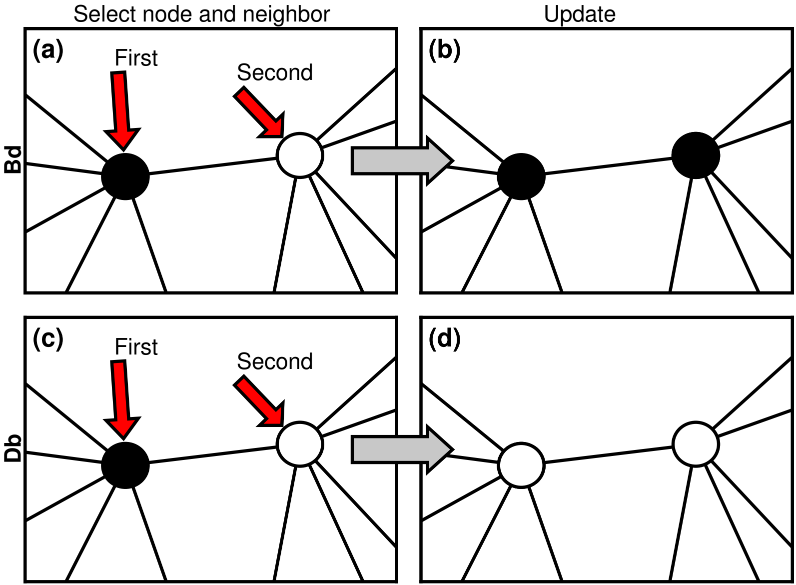
Scientific Reports **7**, Article number: 40360 (2017) | Download Citation

Evolutionary Graph Theory

Definition

The **Moran Birth-death (Bd) model** consists of three steps:
1. With probability proportional to fitness (r), randomly select a node on the network to give birth.
2. Uniformly randomly select a neighbor of the first node to die.
3. The dying node takes on the type of the birthing node.

The Moran Model



Complete Graph: $r = \infty$

Complete Graph: $r = \infty$

At every time step:

1) Select a random node A from the available mutants.
2) Choose random node B from the $N - 1$ neighbors of the first.
3) If B is healthy, turn it into a mutant.
4) Repeat for T steps until every node is a mutant.

Complete Graph: Simplified

At every time step:

1) ~~Select a random node A from the available mutants.~~
1) Choose a random node B from a set of $N-1$.
2) If we haven't seen B before, we label and return it.
3) Repeat for T steps until every node is labeled.

The Coupon Collector's Problem

The Coupon Collector's Problem

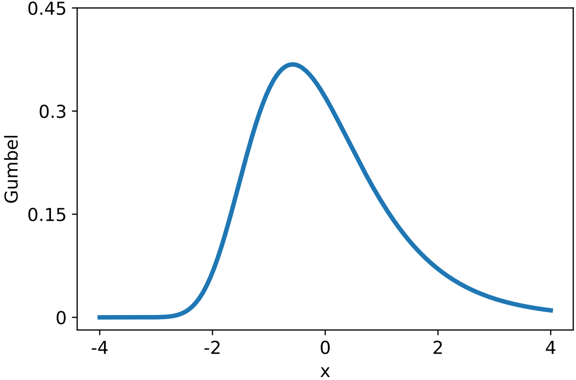
Each day, a kid gets one trading card, uniformly at random. Given that there are N distinct cards, what is the distribution of times T required to form a complete set?

The Complete Graph

Theorem

$$\frac{T - E[T]}{N} \xrightarrow{d} Gumbel(-\gamma, 1),$$

(Where $\gamma =$ the Euler-Mascheroni constant ≈ 0.5772.)



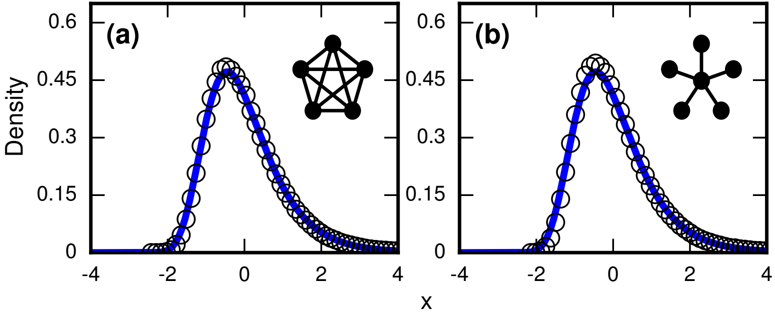
Complete Graph and Star Graph

Complete Graph:

$$\frac{T - N(\log(N) + \gamma)}{N} \xrightarrow{d} \text{Gumbel}(-\gamma, 1). \quad (1)$$

Star Graph:

$$\frac{T - N^2(\log(N) + \gamma - 1)}{N^2} \xrightarrow{d} \text{Gumbel}(-\gamma, 1). \quad (2)$$



Lattices and Moran

Lattices: Surface Area to Volume

To understand lattices, do the following:

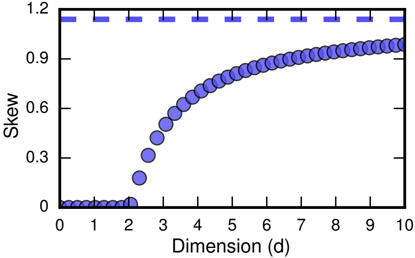
1) Make an analogy to first-passage percolation.
2) Use surface area to volume scaling.
3) Pray.

Lattice: $d \geq 3$.

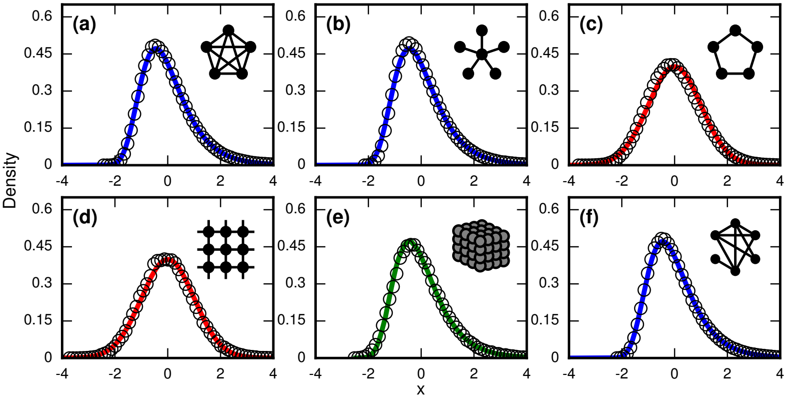
Theorem

Letting $\eta = 1 - 1/d$, the asymptotic skew of the takeover times for a $d > 2$ dimensional lattice is given by

$$Skew(d) = \frac{2\zeta(3\eta)}{\zeta(2\eta)^{3/2}}, \text{ where } \zeta(x) = \sum_{n=1}^{\infty} \frac{1}{n^x}.$$



Results for Infinite r



Non-infinite fitness?

Fitness and Skew for Complete Graph

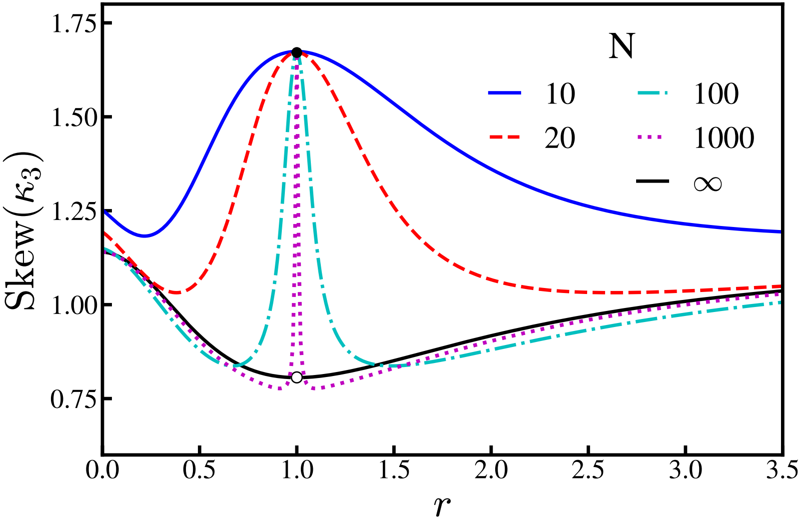


Figure: Hathcock 2018

Realism?

Red = Lognormal, Blue = Gumbel

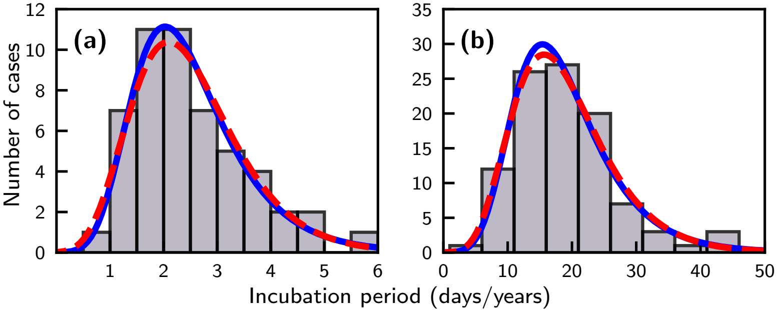


Figure: (a) Food-borne streptococcal sore throat (Sartwell 1950).
(b) Bladder tumors in a dye plant (Goldblatt 1949).

Main Point

Common aspects of disease growth:

- **Evolutionary Network Dynamics**
- **The Coupon Collector's Problem**

Together, they help justify Sartwell's Law.

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Questions?

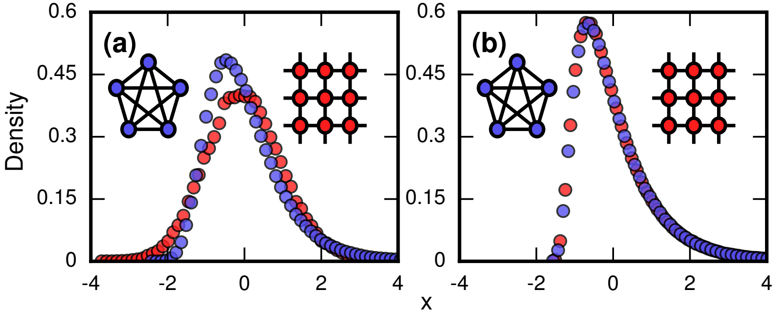

Figure: (a) $r = \infty$. (b) $r = 1$.

All slides available at: ottinoloffler.com